Chapter XII. Special Topics Report
Centrifuge Settling & Filtration Theory

I. Introduction

Settling and filtration are two very important operations for separating solids from a liquid solution. With proper consideration given to flow rates, particle sizes, and system geometry these separation processes can be very selective. Large particles may be trapped in a properly selected filter cloth, while denser particles may accumulate in a sedimentation device. The proper operation of these units requires a basic knowledge of settling and filtration theory. This document attempts to highlight several of the more important equations governing centrifugal settling and filtration theory.

The driving force for settling or filtration has traditionally been due to gravity or pressure differences. However, these forces are sometimes not enough to provide rapid separation. This is because the rate of accumulation of solids follows the general relationship;

Rate = Driving Force / Resistance

This type of relationship appears in many applications including; electrical current, heat transfer, and mass transfer. However, its application to settling and filtration reveals two possible methods for increasing the solid accumulation rate:

1) Increase the Driving Force

2) Decrease the Resistance

Centrifuges use option #1, increasing separation by vastly increasing the driving force. Centrifugal forces are typically several thousand times the force of gravity and can reach up to two hundred fifty thousand g-forces in analytical devices. It is this incredible driving force that makes centrifugal settling such a powerful means of mechanically separating solids.

The following report explores the utility and theory of centrifuges by first examining the nature of the centrifugal force (see section II). This is then applied to the force balance responsible for particle settling (see section III). The Sigma method for scale-up of centrifuges is also briefly examined (see section III.2). Finally, the results of centrifugal forces on the filtration process are presented (see section IV).

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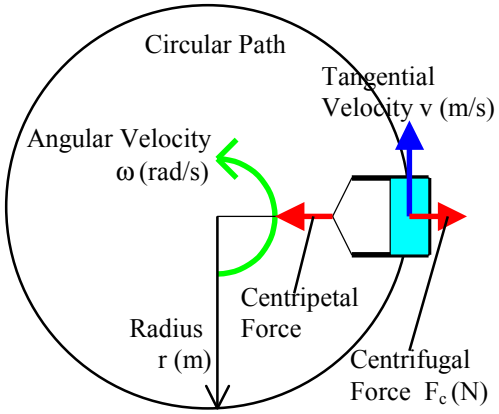
II. Centrifugal Forces

Bodies in circular motion experience an acceleration even though their speed may be constant. This is due to the vector nature of velocity. While the magnitude of the velocity may be constant, its direction is changing (see Figure II-1 below). The magnitude of the velocity for the circling object follows equation II-1.

Figure II-1 **A Bucket filled with Water in Circular Motion**



As the bucket circles, its tangential velocity has a constant magnitude but changing direction. This gives rise to an inward pointing *centripetal* force acting on the bucket, and an equal but opposite outward pointing *centrifugal* force acting on the water. (see Equation II-1,2,3).

$$v_t = \omega r$$

Where: v_t is tangential velocity (m/s)
ω is angular velocity (rad/s)
r is radius of circular path (m)

Equation II-1 Velocity for Circular Motion.

This changing velocity vector gives rise to centripetal and centrifugal forces which are equal in magnitude but opposite in direction. Of these, the centrifugal force is the driving force for centrifuge settling and filtration (see Equation II-2a,b below). Additionally, a centrifugal force is distinguishable from a gravitational force only in magnitude (see equations II-2c&d).

$$a_c = r\,\omega^2$$
$$F_c = m\,a_c$$
$$F_g = m\,g$$
$$RCF = F_c / F_g = r\,\omega^2 / g$$

Where: a_c is acceleration due to centrifugal force (m/s^2)
r is radius of circular path (m)
ω is angular velocity (rad/s)
F_c is centrifugal force (N)
m is mass of the object (kg)
F_g is gravitational force (N)
g is the acceleration of gravity (9.807 m/s^2)
RCF is relative centrifugal force (g force)

Equation II-2 (a,b,c,&d) Centrifugal acceleration and force (G1, p.830) along with gravitational force and relative centrifugal force (P1, p.19-90).

Finally, it should be noted that rotational speeds are often given in units other than radians per second. Such alternates include frequency and revolution per minute, however these can be readily converted into an angular velocity (see equations II-3 a & b below).

$$\omega = 2\pi f$$
$$\omega = (2\pi N) / 60$$

Where: ω is angular velocity (rad/s)
f is frequency (Hz or cycles/sec)
N is revolutions per minute (RPM)

Equation II-3 (a&b) Conversions from common equivalents of angular velocity.

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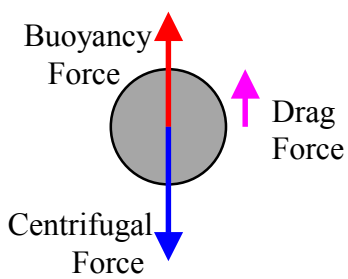
III. Centrifugal Settling

If a solution contains particles with a density greater than that of the liquid, settling will occur. This statement is simply the application of Archimede's principle to microscopic objects. However, the settling time of particles suspended in a solution may be to long to be practical, or Brownian motion and natural convection currents may disturb settling enough that sedimentation will never completely occur under the low driving force of gravity. To circumvent these problems, centrifuge settling provides a relative centrifugal force of several thousand g-force (P1, p19-91). These huge forces can provide for rapid and complete settling and sedimentation.

III.1 Theory

Several forces are important in the settling of particles. These include; gravitational force, centrifugal force, buoyancy, and particle drag. However, for many centrifuges the gravitational force is insignificant compared to the centrifugal force. Gravity will therefor be ignored in this analysis. A balance of the three remaining forces determine the settling velocity of a particle in suspension (see Figure III.1-1).

Figure III.1-1 **Force Balance for Particle Settling**



As the velocity of the falling particle increasing, so will the drag force. When the sum of all three forces equals zero the particle will have reached its settling velocity.

For a continuous flow centrifuge the geometry and flow rate determine if a particle is collected or flows out with the effluent (see Equation III.1-2 on the next page).

To determine the settling velocity a functional form of the drag force must be assumed. For spherical particles with a Reynold's numbers below one Stoke's flow is valid. This leads to an analytical form for the settling velocity (see Equation III.1-1c below).

$$\mathbf{F_c = F_b + F_d}$$

$$\mathbf{F_d = 3\pi\,\mu\,D_p\,v_t}$$

$$\mathbf{v = \frac{D_p^2\,(\rho_p - \rho)\,a_c}{18\,\mu}}$$

Where: $\mathbf{F_c}$ is centrifugal force (N)
$\mathbf{F_b}$ is buoyant force (N)
$\mathbf{F_d}$ is drag force (N)
$\boldsymbol{\mu}$ is viscosity of liquid (kg/m/s)
$\mathbf{D_p}$ is particle diameter (m)
$\mathbf{v}$ is settling velocity (m/s)
$\boldsymbol{\rho_p}$ is particle density (kg/m^3)
$\boldsymbol{\rho}$ is liquid density (kg/m^3)
$\mathbf{a_c}$ is centrifugal acceleration (m/s^2)

Equation III.1-1 (a,b,&c) Force balance used in conjunction with Stoke's flow to determine settling velocity (G1, p.831).

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$$Q_c = \frac{D_p^2 (\rho_p - \rho) V a_c}{9 \mu s}$$

Where: Q_c is volumetric flow rate through the bowl (m^3/s)
μ is viscosity of liquid (kg/m/s)
D_p is particle diameter (m)
ρ_p is particle density (kg/m^3)
ρ is liquid density (kg/m^3)
a_c is centrifugal acceleration (m/s^2)
V is the volume of liquid held in the bowl (m^3)
s is thickness of a thin liquid layer (m)

Equation III.1-2 Settling in a continuous flow bowl centrifuge. Particles with a diameter below D_p will be collected in the centrifuge, while those with a larger diameter will pass through with the effluent. Particles with a diameter of D_p will be equally distributed between the centrifuge and effluent (P1, p.19-95).

Equation III.1-2 is valid for bowl centrifuges with a thin layer of liquid undergoing Stokes settling. When the geometry creates values for the bowl radius and liquid thickness that are not constant, a simple average value will still provide reasonable results. Finally, for situations in which the liquid does not form a thin layer a more complicated relationship must be utilized (see Equation III.1-3 below).

$$Q_c = \frac{D_p^2 (\rho_p - \rho) V \omega^2}{18 \mu \ln[2r_2 / (r_1+r_2)]}$$

Where: Q_c is volumetric flow rate through the bowl (m^3/s)
μ is viscosity of liquid (kg/m/s)
D_p is particle diameter (m)
ρ_p is particle density (kg/m^3)
ρ is liquid density (kg/m^3)
ω is angular velocity (rad/s)
V is the volume of liquid held in the bowl (m^3)
r_1 is the distance from the center to the liquid layer (m)
r_2 is the distance from the center to the centrifuge wall (m)

Equation III.1-3 Settling in a continuous flow bowl centrifuge. Particles with a diameter below D_p will be collected in the centrifuge, while those with a larger diameter will pass through with the effluent. Particles with a diameter of D_p will be equally distributed between the centrifuge and effluent (G1, p.832).

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III.2 Sigma Theory and Scale-Up

To simplify scale-up calculations for centrifuges it is desirable to have the centrifuge properties separated from the particle properties. Sigma theory makes this desired characteristic a reality by describing a centrifuge's characteristics in terms of an equivalent sedimentation process that would be conducted with gravity alone (see Equation III.2-1a&b).

$$Qc = 2\, v\, \Sigma$$
$$\Sigma = 2\pi\, \omega^2\, b\, r_2^2 / g$$

Where: $\mathbf{Q_c}$ is volumetric flow rate through the bowl (m^3/s)
- $\mathbf{v}$ is the settling velocity (m/s)
- $\boldsymbol{\omega}$ is angular velocity (rad/s)
- $\mathbf{b}$ is the length of the centrifuge wall along which the fluid flows (m)
- $\mathbf{r_2}$ is the distance from the center to the centrifuge wall (m)
- $\boldsymbol{\Sigma}$ is a physical characteristic of the centrifuge only. It represents the area a gravitational settler would have to be to have the same characteristics as the centrifuge. (m^2)
- $\mathbf{g}$ is the acceleration of gravity (9.807 m/s^2)

Equation III.2-1 (a&b) Equations for sigma theory separate the equations governing centrifugal settling into two parts relating to the particle and the centrifuge. The sigma value represents the area a gravitational settler would need to be to have the same characteristics as the centrifuge in question (G1, p.834).

$$Q_1 / (\Sigma_1 E_1) = Q_2 / (\Sigma_2 E_2)$$

Where: $\mathbf{Q_1}$ is volumetric flow rate of centrifuge #1 (m3/s)
- $\boldsymbol{\Sigma_1}$ is the sigma value for centrifuge #1 (m2)
- $\mathbf{E_1}$ is the efficiency factor of centrifuge #1 (Dimless)
- $\mathbf{Q_2}$ is volumetric flow rate of centrifuge #2 (m3/s)
- $\boldsymbol{\Sigma_2}$ is the sigma value for centrifuge #2 (m2)
- $\mathbf{E_2}$ is the efficiency factor of centrifuge #2 (Dimless)

Equation III.2-2 Scale-up relationships for centrifuges using sigma values (G1, p.834).

Once sigma values have been calculated for the centrifuge under investigation, the theory provides a reliable method for proper scale-up (see Equation III.2-2 on left).

When the centrifuges under consideration have a similar geometry and also have centrifugal forces within a factor of two, the efficiency factors may be set equal and canceled out of the equation. Otherwise, a table of efficiency factors should be consulted (see P1, p.19-96 for such a table).

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IV. Centrifugal Filtration

Filtration is carried out by supplying a pressure gradient across a filter cloth. This pressure difference forces the liquid through a porous cake and cloth where the particles are trapped. Traditionally this driving force is generated by gravity and a partial vacuum on the other side of the cloth. Centrifugal forces provide an alternate method capable of generating very high pressure gradients across the filter cloth.

IV-1 Theory

$$Q = \frac{\rho\, \omega^2 (r_2^2 - r_1^2)}{2\mu\, (m_c\, \alpha / A^2 + R_m / A)}$$

Where: **Q** is volumetric flow rate through filter (m^3/s)
- **ρ** is the density of the filtrate (kg/m^3)
- **ω** is angular velocity (rad/s)
- $\mathbf{r_1}$ is the distance from the center to the cake surface (m)
- $\mathbf{r_2}$ is the distance from the center to the centrifuge wall (m)
- **μ** is the viscosity of the solution (kg/m/s)
- $\mathbf{m_c}$ is mass of cake deposited on filter (kg)
- **α** is specific cake resistance (m/kg)
- **A** is area of cake (m^2)
- $\mathbf{R_m}$ is resistance of the filter medium to filtrate flow (1/m)

Equation IV.1-1 Equation for centrifugal filtration. If the area "A" varies with radius the relationship must be altered as described in G1, p.838. The "A^2" term becomes the product of the logarithmic cake area and the arithmetic mean cake area, while the "A" term becomes the area of filter medium. (G1, p.838).

Filtration centrifuges are sometimes referred to by other names such as; wringers, extractors, or dryers. However, they all have the same purpose; the removal of a solid from a liquid solution by passing the solution through a bed of solids held on a screen. Whether the centrifuge operates in a batch, semi-batch, or continuous mode makes little deference to the theory of operation (see Equation IV.1-1 on left).

V. References

(G1) Geankoplis, Christie J. *Transport Processes and Unit Operations*, 3rd ed. Englewood Cliffs, New Jersey: Prentice Hall PTR, 1978.

(P1) Perry, R. H., and Green, D. *Perry's Chemical Engineers' Handbook*, 6th ed. New York: McGraw-Hill Book Company, 1984.

(S1) Streeter, Victor L., and Wylie, Benjamin E. *Fluid Mechanics*, 8th ed. New York: McGraw-Hill Book Company, 1986.

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