Logarithmic Graphs

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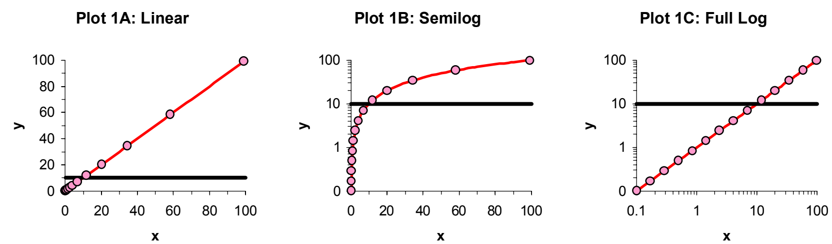
Table 1: Logarithm Bases

Original Number	Scientific Notation	Base 10	Base 'e' (2.718...)	Base 2
x	x	$\log_{10}(x)$	ln(x)	$\log_2(x)$
1,000	1×10^3	3	6.908	9.966
100	1×10^2	2	4.605	6.644
10	1×10^1	1	2.303	3.322
8	8	0.903	2.079	3
5	5	0.699	1.609	2.322
4	4	0.602	1.386	2
3.162	3.162	0.500	1.151	1.661
2.718	2.718	0.434	1	1.443
2	2	0.301	0.693	1
1.414	1.414	0.151	0.347	0.5
1	1	0	0	0
0.5	5×10^{-1}	-0.301	-0.693	-1
0.1	1×10^{-1}	-1	-2.303	-3.322
0.01	1×10^{-2}	-2	-4.605	-6.644
0.001	1×10^{-3}	-3	-6.908	-9.966
0	0	$-\infty$	$-\infty$	$-\infty$
-1	-1	$0.434\,\pi\,i$	$\pi\,i$	$1.443\,\pi\,i$
-10	-1×10^1	$1 + 0.434\,\pi\,i$	$2.303 + \pi\,i$	$3.322 + 1.443\,\pi\,i$
100 sec	1×10^3 sec	2 + $\log_{10}$(sec)	4.605 + ln(sec)	6.644 + $\log_2$(sec)

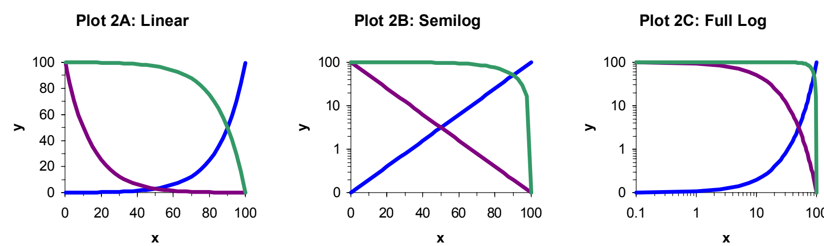
A few numbers and their logarithms. Note that small numbers have negative logarithms and the log of zero is minus infinity. Also, values plotted at zero on a logrithmic scale are really equal to one, not zero. Furthermore, to convert from one base to another simply multiply by a common factor. To go from base 10 to base e multiply all values by 2.303 (i.e. ln(10)). Similarly, multiply all values by 3.322 (i.e. $\log_2(10)$) to go from base 10 to base 2 and 1.443 (i.e. $\log_2(e)$) from base e to base 2. Additionally, negative numbers have imaginary logarithms (because $e^{-\pi i}$ = -1). Lastely, logarithms of values with units should be written as the log of the value PLUS the log of the unit (e.g. log(2m) = log(2) + log(m)).

Plotting With Logarithms: A Few Examples

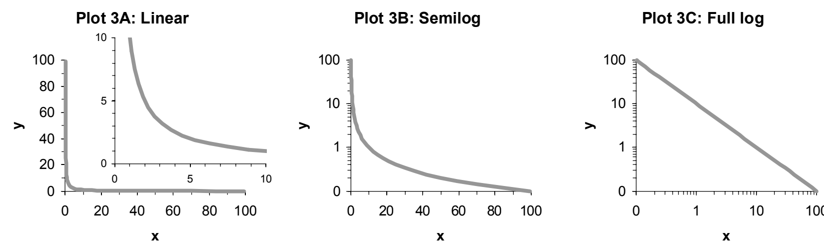
Plots 1A, 1B, & 1C show that linear data becomes curved when plotted on semilog scales (unless it has a zero slope). However, full log scales keep straight lines straight. They are mainly used to increase resolution by separating bunched up data points.



Plots 2A, 2B, & 2C show that exponentially increasing data becomes staight when plotted on semilog scales (blue line). Exponentially decreasing data is also straightened (purple lin). However, not all data containing an exponential is straightened (green line). Even full log scales do not straighten this data.



Plots 3A, 3B, & 3C show data that is exponential with respect to both the x & y axis (very rarely seen in nature). Full log scales straighten out this type of data.



Plots 4A & 4B show why log graphs are sometimes called "ratio" or "rate-of-change" graphs. The first four lines show values that increase 10 fold (i.e. 0.1 to 1, 1 to 10, 2 to 20, and 5 to 50). When plotted on semilog scales these lines are all the same length. The same thing happens at 50 fold.